INDIAN STATISTICAL INSTITUTE
CHENNAI CENTRE
M.STAT I. 20116-17 Semester II
Multivariate Analysis
Mid-Semester Examination

Date : 24.02.2017 **Time :** 2 **hours**

This paper carries 75 marks. Answer as much as you can. Maximum you can score is 70. Marks are in [ ] and the end of each question. Do state the Theorems you are using in deriving your answers.

1. Suppose $\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_n$ are iid $N_p(\mathbf{0}, \Sigma)$. Find the MLE of Σ. Show that it is an unbiased estimator of Σ. [10+10=20]

   Hint : Take a p-vector $\mathbf{a} \neq 0$. Define $\mathbf{y} = \mathbf{a}'\mathbf{x}$. Find MLE for variance of $\mathbf{y}$ and work backwards.

2. Let $\mathbf{X}_{n\times p}$ be a data matrix from a p-variate $\mathbf{x} \sim N_p(\mu, \Sigma)$, and $P_{n\times n}$ be an orthogonal matrix with the last row $\frac{1}{\sqrt{n}}$ times the unit vector. Suppose $\mathbf{Y} = P.\mathbf{X}$. Denote the rows of $\mathbf{Y}$ by $\mathbf{Y}_1, \mathbf{Y}_2, \ldots \mathbf{Y}_n$. Show that

   (a) $\mathbf{Y}_i$, $i \in \{1, 2, \ldots n\}$ are independent.

   (b) $\mathbf{Y}_n$ follows $N_p(\sqrt{n}\mu, \Sigma)$

   (c) $\mathbf{Y}_i$, $i \in \{1, 2, \ldots n-1\}$ are i.i.d. $N_p(\mathbf{0}, \Sigma)$

   (d) Express $\bar{\mathbf{x}}$ in terms of $\mathbf{Y}_n$ only, and $\mathbf{A}$, the sum of square & sum of products matrix of the p-variate $\mathbf{x}$ in terms of $\mathbf{Y}_i$, $i \in \{1, 2, \ldots n-1\}$

   (e) State the distributions of $\bar{\mathbf{x}}$ and $\mathbf{A}$, and comment on their dependence.[4+4+6+5+6=25]

3. Let $X \sim N_2\left(\begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\right)$ and

   $Y|X \sim N_2\left(\begin{bmatrix} X_1 \\ X_1 + X_2 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right)$

   (a) Determine the distribution of $Y_2|Y_1$.

   (b) Determine the distribution of $W = X - Y$. [10+5=15]

4. Find the mean vector and the dispersion matrix of the random vector $X = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}$ with joint p.d.f. [5+10=15]

$$f(x_1, x_2) = \frac{1}{2\pi} exp\left[-\frac{1}{2}\{2x_1^2 + 5x_2^2 - 6x_1x_2 - 54x_1 + 84x_2 - 369\}\right]$$